1. I run a pizza restaurant that is only open on Thursday, Friday and Saturday evenings from 5:30 to 9:30. Over that time, orders for pizza come into the restaurant randomly with a distribution that is approximately normal with mean 20 and standard deviation of 5 pizzas per hour. A pizza requires 1 kg of dough, 250 g of cheese and a mix of other ingredients depending on the order.

On Thursday AM I have 100 kg of dough on hand and am placing an order for more dough. The order will arrive the morning of the next day (Friday) on a truck from the bakery. This order is intended to last two weeks when the next order will arrive, also on a Friday morning.

* 1. How likely is it that I will run out of dough before the end of day Thursday?

If we consider the dough inventory alone, we can see from the ingredients of pizza that one order of pizza required 1kg of dough.

A batch of new dough supply would come the next morning, so the inventory of dough on Thursday AM only needs to satisfy the requirement of Thursday’s orders. Since orders for pizza come into the restaurant randomly with a distribution that is approximately normal with mean 20 and standard deviation of 5 pizzas per hour. The dough requirement for Thursday can be described by X ~ Normal(20\*4, 5\*sqrt(4)) = Normal(80,10) in kg.

The event of dough shortage would be X>100. P(X>100) = 1 – P(X<100) =1-NORM.DIST(100,80,5\*2,1) = 2.28%, so that is very unlikely.

However, this conclusion needs to be qualified with the assumption that this Thursday is just one of the ordinary operating days. If some extraordinary events occurred, say sports event, public interest in our pizza spiked, then such information needs to be incorporated into the modeling process.

Also we are estimating a quantity that is aggregate of the hourly dough requirements, the implicit assumption is that the hourly needs are independent of each other, if this assumption do not hold, the analysis needs to be re-worked.

* 1. Suppose that I actually have 200kg of dough on hand on the start of day Thursday. How much dough should I order to have no more than a 1% chance of stocking out by the time the next delivery arrives? (With 200kg of dough on hand, you can assume you make it through Thursday.)

A delivery of dough is supposed to last two weeks and this Thursday, or 7 operating days.

The dough requirement in kg can be described as X ~ Normal(20\*7\*4, 5 \* sqrt(7\*4)) = Normal(560, 26.46). to prevent the extreme situation of outage, we need to look at an threshold value Y, where P(X>Y) = 0.01.

We can work out Y =NORM.INV(1-0.01,560,26.46) = 621.55. since we already have 200kg on hand, the next order should be around 621.55 -200 = 421.55, or 422kg of resupply of dough.

* 1. Not many customers order double cheese. In fact, of the last 49 pizzas that were ordered, only 7 had double cheese. With this information, calculate the 95 % confidence interval on the proportion of customers who order double cheese.

This relates to the proportions, with the latest data on hand, we can have a point estimate the proportion as p = 7/49 = 14.28%. A side note, using the last orders for estimation does not mean these fit the random sampling criterion, so the aforementioned and below analysis needs to be taken with a grain of salt. It can be easily constructed that people may prefer cheesy pizza when it is getting late, and we would be over estimating.

For a pilot analysis, I shall suppress these doubts. Standard error of the proportions can be calculated as se = sqrt(p\*(1-p)/n) = 5.00% (n=49, p = 1/7). To create the confidence interval, with Z\_95% =-NORM.S.INV((1-95%)/2) = 1.96. then the interval would be [4.49%, 24.08%].